

THE COUPLED TURBULENT TRANSPORTS OF SALT AND HEAT ACROSS A SHARP DENSITY INTERFACE

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(Received 19 August 1964 and in revised form 3 December 1964)

Abstract—When a layer of cold fresh water is resting on top of hot salty water with a sharp interface between them, the system as a whole may be unstable, even though the differences of density $\beta\Delta S$ due to salinity are far greater than those due to temperature $\alpha\Delta T$. Convective motions are produced in both layers by the transfer of heat, and at the same time salt is transported across the interface. Measurements are presented here of the rates of transport in such a two-layer system in which the temperature difference is maintained by heating continuously from below.

Both the heat transfer and the ratio of the rates of turbulent transport of salt and heat have been found to depend systematically on the density ratio $\beta\Delta S/\alpha\Delta T$, at least over the small range of heat fluxes used in these experiments. This is in agreement with the result of a simple dimensional analysis, and suggests that the form of the functions should have a wider application. The measurements also indicate that over a wide range of density differences the potential energy change of the top layer due to the lifting of salt is a constant fraction of that released by the transfer of heat.

NOMENCLATURE

$A = ck_T \left(\frac{g\alpha}{\kappa\nu}\right)^{\frac{1}{2}}$	a factor in the heat flux equation, which depends on temperature;
c ,	a numerical constant with the experimental value ≈ 0.085 ;
f, f_1, f_2, f_3 ,	non-dimensional functions;
F_S ,	flux of salt through the interface;
$\frac{F_S}{k_S\Delta S/h}$	non-dimensional salt flux (equivalent to a Nusselt number for salt);
g ,	acceleration due to gravity;
h ,	layer depth;
H ,	flux of heat through the interface;
k_S, k_T ,	molecular diffusivities for salt and heat;
K_S, K_T ,	effective turbulent transfer coefficients for salt and heat;
$Nu = \frac{H}{k_T\Delta T/h}$	the Nusselt number, or non-dimensional heat flux;
$Ra = \left(\frac{g\alpha}{\kappa\nu}\right)\Delta T h^3$	Rayleigh number.

Greek symbols

α ,	coefficient of volume expansion due to unit temperature change;
β ,	the proportional density change produced by unit salinity change;
$\alpha\Delta T$,	density difference due to the temperature difference between layers;
$\beta\Delta S$,	density difference due to the salinity difference between layers;
$\frac{\beta F_S}{\alpha H}$,	the ratio of the potential energy change in the top layer due to lifting of salt, to that due to the transfer of heat;
$\Delta\rho$,	net density difference between layers;
ΔS ,	salinity difference between layers;
ΔT ,	temperature difference between the centres of the layers, by symmetry equal

κ , to that between two rigid
 walls bounding each layer;
 ν , thermometric conductivity;
 kinematic viscosity.

INTRODUCTION

THE STRATIFICATION of an initially smooth, stable distribution of salinity caused by heating from below has been described elsewhere [1]. The heating of the bottom of a column stabilized with salt produces convection locally, but the penetration is limited by the salt, because it diffuses more slowly than heat. The combined effect of heat and salt is to produce a series of turbulent layers which are well mixed in both salinity and temperature, but separated by sharp interfaces from their neighbors. A photograph of a typical experiment is shown in Fig. 1. In experiments of this kind it is observed that the lowest two layers will after a time merge, so that the thickness of the bottom layer is increasing in steps. This implies that the densities of these merging layers are coming closer together, and that there is a transfer of salt as well as heat across the interface, at a rate which is much greater than molecular diffusion can account for. This is also made obvious by the behavior of dye injected into the lowest layer, some of which is quickly transferred into the successive layers above.

There are many questions raised by these seemingly simple experiments. Eventually one might hope to understand how the layer depth depends on the salinity gradient and on the rate of heating, and to evaluate the importance of the mechanism for vertical transports in the ocean. The first step, however, seems to be to learn something about the transfer processes at *single* density interface, such as that shown in Fig. 2, and this is the problem with which we will be concerned here. How are the relative rates of transport of heat and salt affected by the stability of the interface, and the fact that the fluid is in turbulent convective motion on both sides of it? The level of turbulence at such an interface can of course depend both on the convective motions produced by local temperature contrasts, and on convective (or mechanical) turbulence produced elsewhere in the layer. Ideally we should study one of a number of identical layers, having fluid

interfaces above and below and maintained in a steady state by heating and supplying salt below and cooling and removing salt above at the same rates. This is not simple to do, and in this exploratory investigation it was not attempted, since it was convenient to use the change of properties in the upper layer as a measure of the fluxes.

At large rates of heating and small layer depths, it is apparent that the convective elements formed at the bottom have a dominant effect on the mixing at the interface above. In the experiments to be reported here, we have restricted ourselves to deep layers and low rates of heating in the hope that the motion near the interface will be governed mostly by the temperature and salinity contrasts across it. This idea receives support from the observation that under these circumstances the position of this boundary moves very slowly or not at all before it breaks down completely. Thus the rate of transport of matter is the same in both directions, and this is likely to be so only if the level of turbulence on the two sides is nearly the same, and dominated by the properties of the interface itself. The real test of the validity of this assumption will be our ability to explain the experimental data in terms of the properties of the interface alone, and it does seem to be justified by the results reported in this paper.

DESCRIPTION OF THE EXPERIMENTS

The experimental procedure used was the following. Two layers of common salt solution, each 12 cm thick, were set up in a cylindrical perspex tank 30 cm in diameter. The interface was made as sharp as possible by stirring mechanically from both sides or by inserting a thin sheet while filling, and the initial density difference was varied from less than one part per thousand (0.001 g/cm^3) up to about 10 per cent (0.1 g/cm^3). The bottom of the tank consisted of a metal plate, and this rested on an electrically heated pad to provide uniform heating over the whole area of the bottom. Except for the purposes of photography, the side walls of the tank were insulated with a thick plastic foam jacket to reduce heat losses, and the top water surface was also covered with a rigid, insulating lid. The general arrangement of the apparatus is shown in Fig. 3.

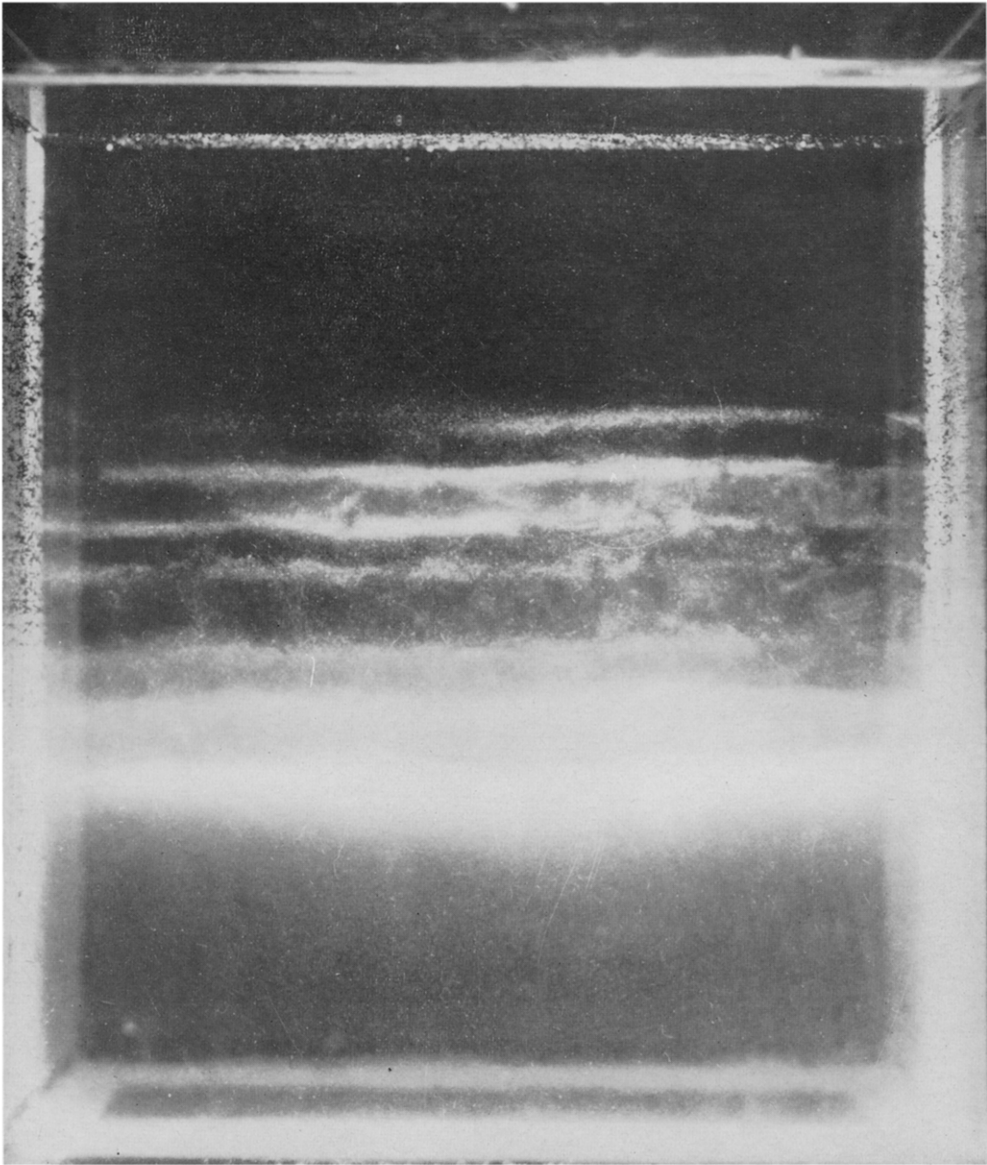


FIG. 1. Showing the many layers formed when a tank containing a linear salt distribution is heated from below.

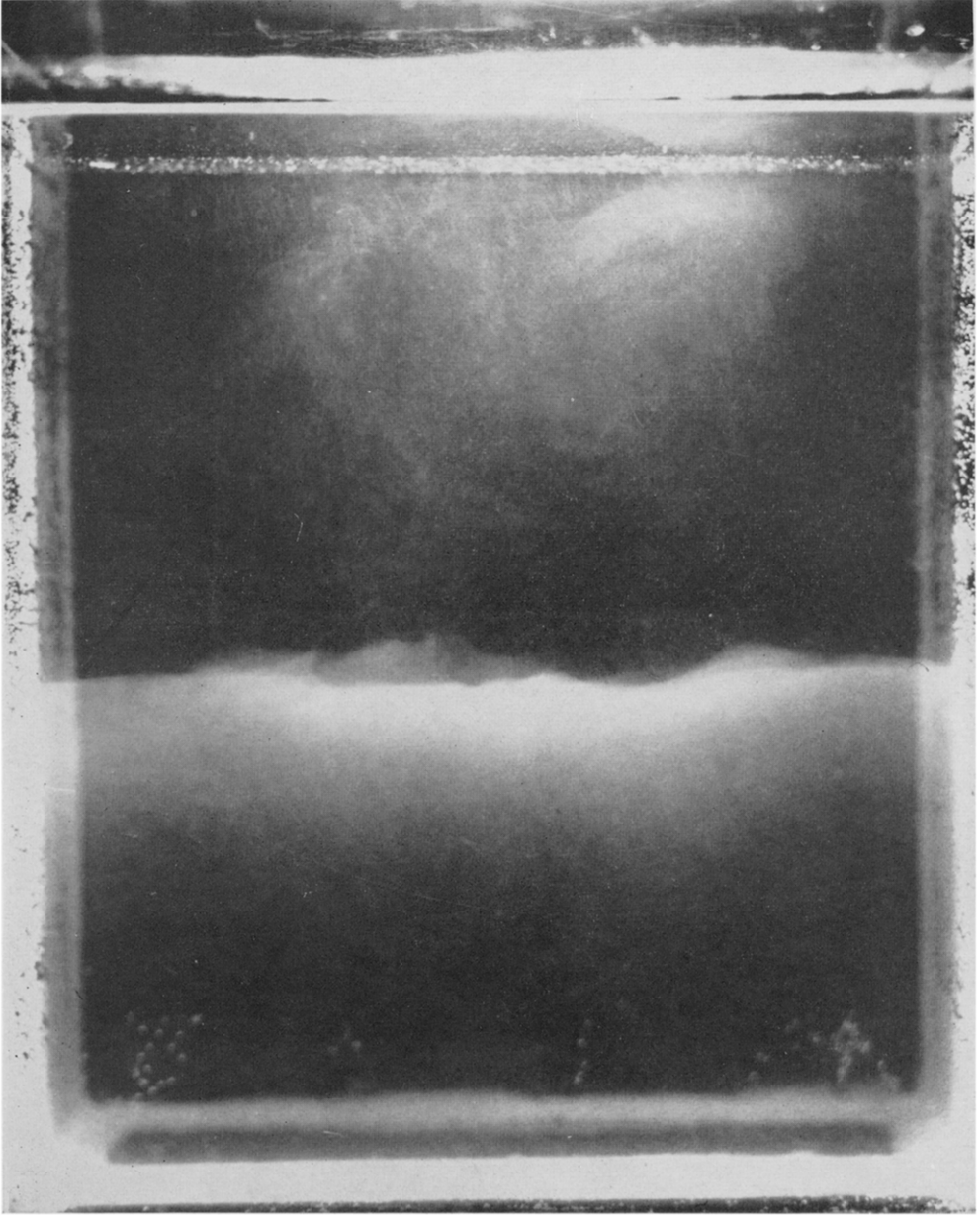


FIG. 2. The two-layer system studied in the experiments reported in this paper.
The lower, more saline layer has been coloured with fluorescent dye.

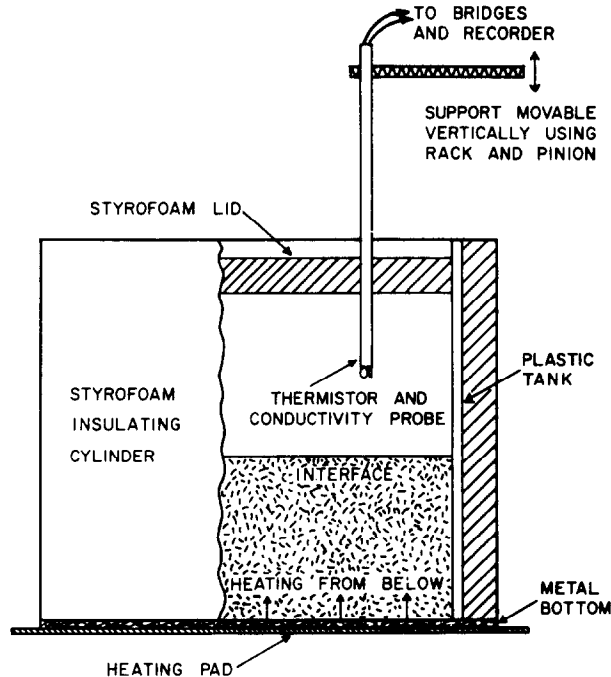


FIG. 3. A schematic diagram of the experimental arrangement.

Initially there was no temperature difference between the layers, and therefore no source of energy to drive any motion. The necessary temperature contrast was produced by heating gently from the bottom. First the lower layer began to convect, and then the upper as heat was transferred through the interface. At this stage some further mechanical mixing was often necessary, before convection of the upper layer was established. A third, intermediate layer can form in the thin stable region between the deep layers, and this can inhibit the transfer of heat. All the experiments reported here were begun with sharp interfaces and a strictly two-layer system, although with large density differences an intermediate region builds up later as salt is spread out by the convective motions.

In order to calculate the transfers of heat and salt across the interface, the raw data required are the temperatures and salinities of the two layers as a function of time. These were measured using a combined thermistor and conductivity probe, with the inputs fed into bridge circuits and the output displayed on an X - Y recorder

which plots temperature against conductivity. The probe was calibrated directly against temperature and salinity, using standard solutions of known concentration. In exploratory runs, the probe was traversed across each layer and the variations of temperature and salinity recorded. These were found to be small within each layer, and of the same order as the fluctuations, so for most of the measurements the probe was alternated between the centres of the two layers and the measurements taken there. Typically, successive pairs of measurements were 5–15 min apart, with an averaging time of about $\frac{1}{2}$ min at each position.

During the course of a run, the temperatures of both layers increased continuously while the salinities changed in opposite directions because the total amount of salt in the system remained constant. Since the salt used was weighed, the whole of the information about salt diffusion could be obtained from measurements of the increase in salinity in the top layer alone. The salinity of the lower layer was used only as a check, and at the later stages of an experiment

(especially at the smallest density differences) the probe was left in the upper layer, to avoid disturbance of the interface. The temperature of the bottom layer was at these times read directly with a bulb thermometer.

THEORETICAL CONSIDERATIONS

The interpretation of measurements made in this way, and indeed the best method of presentation, depend on the mechanism of the transfer process, which we do not know in advance. In this section we shall anticipate some of the experimental results, and discuss several theoretical ideas on which we have based a dimensional argument. This will make it easier to appreciate the choice of parameters and the implications of the data presented in the following section.

Since the convective motions in the two layers are being driven by the temperature difference across the interface, a useful question to ask first is: how is the heat transport changed by the nature of the interface, compared with, say, the corresponding flux between two solid planes across which the same temperature difference is maintained? More specifically, let us suppose that a thin rigid sheet of a perfect conductor is inserted at the interface. This prevents wave-like motions and the diffusion of salt, but is kept at the same temperature as before (the mean temperature of the layers).

The turbulent heat flux into the deep layer on either side of such a plane will be described by

$$Nu = cRa^{\frac{1}{3}}. \quad (1)$$

This is the appropriate form at large Rayleigh number since it implies that the layer depth does not enter explicitly into the determination of heat flux. The constant c has been obtained experimentally by several workers: the best value appears to be 0.085 (Chandrasekhar [2]), though slightly higher values have been suggested. (See, for example, the discussion by Herring [3].) Using the definitions of Nu and Ra , we can write (1) as

$$H = A(\Delta T)^{4/3} \quad (2)$$

where

$$A = ck_T \left(\frac{g\alpha}{\kappa\nu} \right)^{\frac{1}{3}} \quad (3)$$

is a factor which is calculable for any measured conditions of temperature and salinity.

When the interface is able to move and salt is being transferred, the Nusselt number must again be a function of the Rayleigh number based on the temperature difference. It will now, however, also depend on the diffusion coefficient for salt, and on the density difference, or to be more specific, on the relative contributions $\alpha\Delta T$ and $\beta\Delta S$ of temperature and salinity differences to the density (where α and β are the "coefficients of expansion" due to unit temperature and salinity changes). Dimensional arguments suggest that

$$Nu = f(Ra, \beta\Delta S/\alpha\Delta T, k_S/k_T).$$

If we choose the Ra dependence so that the heat transfer is again independent of depth, and omit the ratio of molecular diffusivities since this remains nearly constant in all the experiments reported here, then a more explicit equation can be written:

$$Nu = f_1 \left(\frac{\beta\Delta S}{\alpha\Delta T} \right) Ra^{\frac{1}{3}}. \quad (4)$$

Here

$$f_1(\beta\Delta S/\alpha\Delta T)$$

is a function which must be determined experimentally; as we shall see, this form does receive good support from our measurements.

Now let us consider the flux of salt. It is the unstable temperature distribution which is driving the whole motion, and therefore is ultimately responsible for mixing salt across the interface. It therefore seems reasonable to look again for a dependence of the (non-dimensional) salt flux on the Rayleigh number formed from the *temperature* difference, and on the stability properties of the interface. Using the same arguments as led to (4), the dimensionless salt flux will be of the form

$$\frac{F_S}{k_S \Delta S/h} = f_2 \left(\frac{\beta\Delta S}{\alpha\Delta T} \right) Ra^{\frac{1}{3}}. \quad (5)$$

This special functional form is again chosen to eliminate the explicit dependence on h . Taking the ratio of (4) and (5) we see that

$$\frac{F_S}{\Delta S} \cdot \frac{\Delta T}{H} = \frac{k_S}{k_T} f_3 \left(\frac{\beta\Delta S}{\alpha\Delta T} \right). \quad (6)$$

The left-hand side of (6) may be written as the ratio K_S/K_T of turbulent transfer coefficients K_S for salt and K_T for heat across the interfacial

layer. The function f_3 in (6) is a measure of the deviations of this ratio from the molecular value k_S/k_T . Again, the experiments will give support to this kind of relation between K_S/K_T and the density ratio.

THE MEASUREMENTS OF HEAT AND SALT FLUX

The experimental results will now be presented, within the framework of the ideas discussed above.

The basic measurements of temperature and salinity were used as follows. The fluxes of heat and salt H and F_S were deduced from the changes in temperature and salinity of the upper layer over a convenient interval of time (which was varied according to the density difference). The differences ΔT and ΔS between the layers were averaged over this interval, and the contributions of each to density, $\alpha\Delta T$ and $\beta\Delta S$, were found from tables, taking into account of course the variations of α and β with temperature

and salinity. Experiments begun with a small salinity difference between the layers were usually followed until the layers overturned and mixed completely, and several measurements over a range of density differences were obtained during the same run. With larger salinities, it was found convenient to cover a small range each time, and to start again rather than letting the mean temperature build up to high values. Because of the limitations discussed in the first section, the rates of heating from the bottom which could conveniently be used were rather restricted and covered a range of only a factor of 3. The Rayleigh number in our experiments was of order 10^8 .

The heat flux measurements are presented in Fig. 4 as a function of the density ratio. The values obtained directly have been compared with the "solid wall" values calculated according to (2) and (3) for the same temperature difference. (The effect of temperature on the molecular

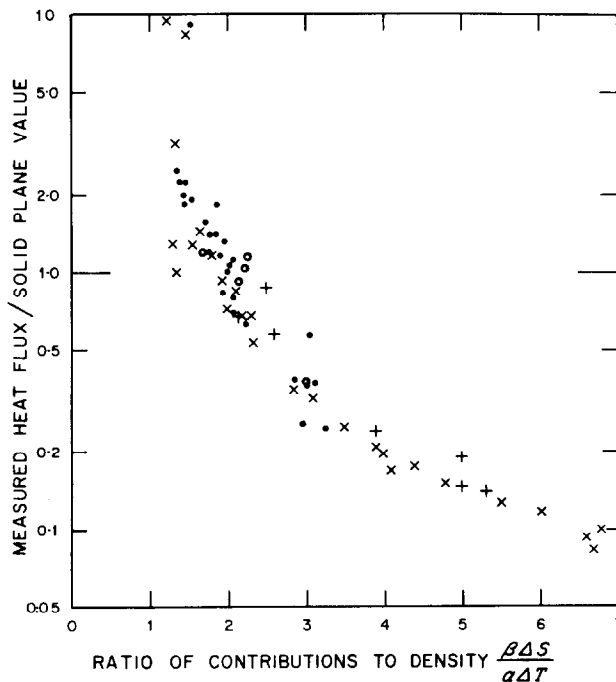


FIG. 4. The measured heat flux across the interface between the two liquid layers, compared with the value calculated for solid planes. This ratio has been plotted against the stability parameter $\beta\Delta S/\alpha\Delta T$, the ratio of the contributions of salt and heat to the density difference. In this and the following figures, different symbols have been used for experiments with different heating rates:

○ 1.1 cal/cm²/min; ● 1.7 cal/cm²/min; + 2.4 cal/cm²/min; × 3.3 cal/cm²/min.

properties in A has been taken into account, but not the smaller changes due to salinity). The estimated error of individual measurements, based on the accuracy of temperature and salinity determinations, is about 30 per cent at the smaller values of $\beta\Delta S/\alpha\Delta T$ and rather better at the higher end. In spite of this large scatter the measurements are still useful because of the 100:1 variation over the range measured. The data are consistent with a relation of the form (4), but detailed discussion will be deferred till the next section.

Figure 5 shows the corresponding plot of the ratio of the transfer coefficients for salt and heat K_S/K_T against $\beta\Delta S/\alpha\Delta T$. Again the data seem to be organized according to the suggested relation (6), with little dependence on the heating rate.

In Fig. 6 the ratio $\beta\Delta S/\alpha\Delta T$ has been plotted against the net density difference between the layers, in order to show that there is a strong correlation between these two quantities (with perhaps more systematic dependence on the rate of heating than is apparent in Fig. 4 and 5). This is a rather surprising result which is unlikely to remain generally valid over a large range of

heating rates, or in experiments in which the temperature difference rather than the heat flux is specified. It does mean, however, that for the limited range of heat fluxes used in these experiments, Figs. 4 and 5 could equally well be plotted against $\Delta\rho$ instead of the density ratio.

Another quantity which will be required in the following discussion is the ratio

$$K_S \beta\Delta S / K_T \alpha\Delta T = \beta F_S / \alpha H,$$

which may be interpreted as the ratio of the potential energy change of the top layer due to the lifting of salt, to that due to the transfer of heat over the same interval of time. This is shown in Fig. 7 as a function of $\beta\Delta S/\alpha\Delta T$.

DISCUSSION

The good organization of the experimental data has given support to the dimensional arguments outlined above, and leads to the conclusions that both the heat flux and the relative rates of transfer of salt and heat, through a stable interface heated from below, depend systematically on the stability of the

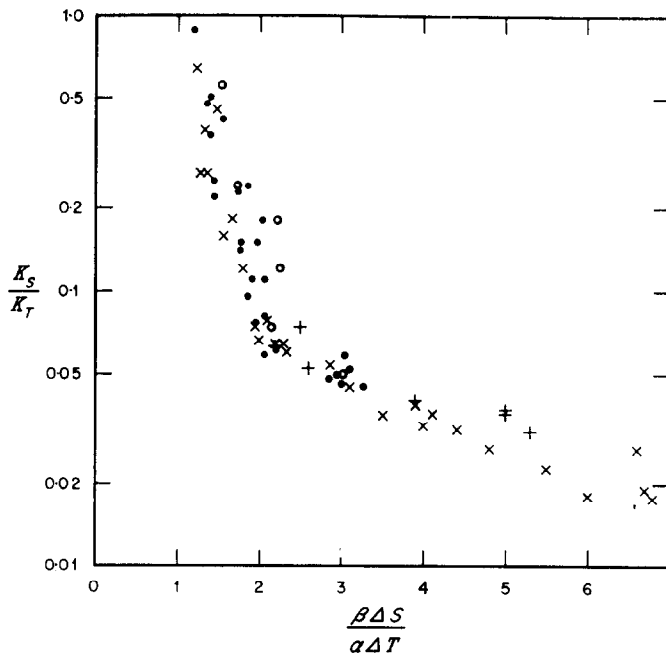


FIG. 5. The ratio of the turbulent transfer coefficients for salt and heat, plotted as a function of the stability parameter $\beta\Delta S/\alpha\Delta T$.

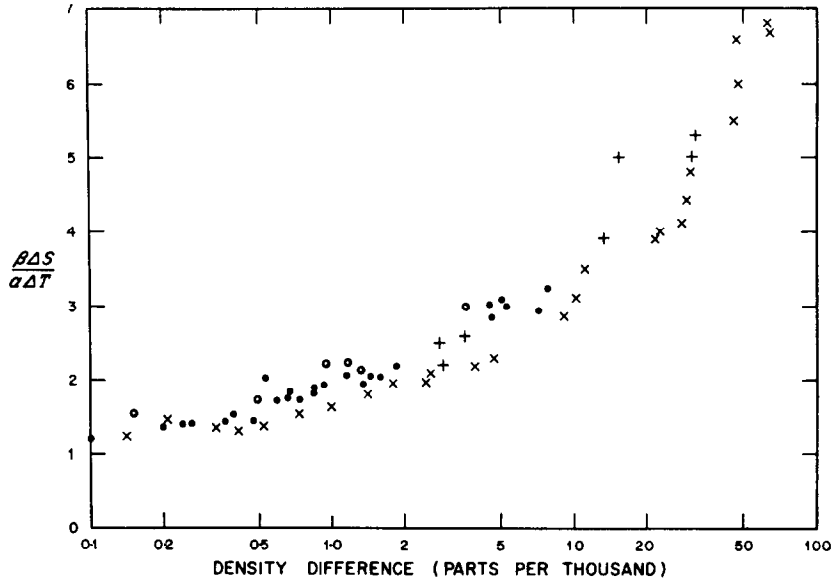


FIG. 6. The ratio $\beta \Delta S / \alpha \Delta T$ plotted against the net density difference between the two layers. It is suggested that this relation will be sensitive to the rate of heating.

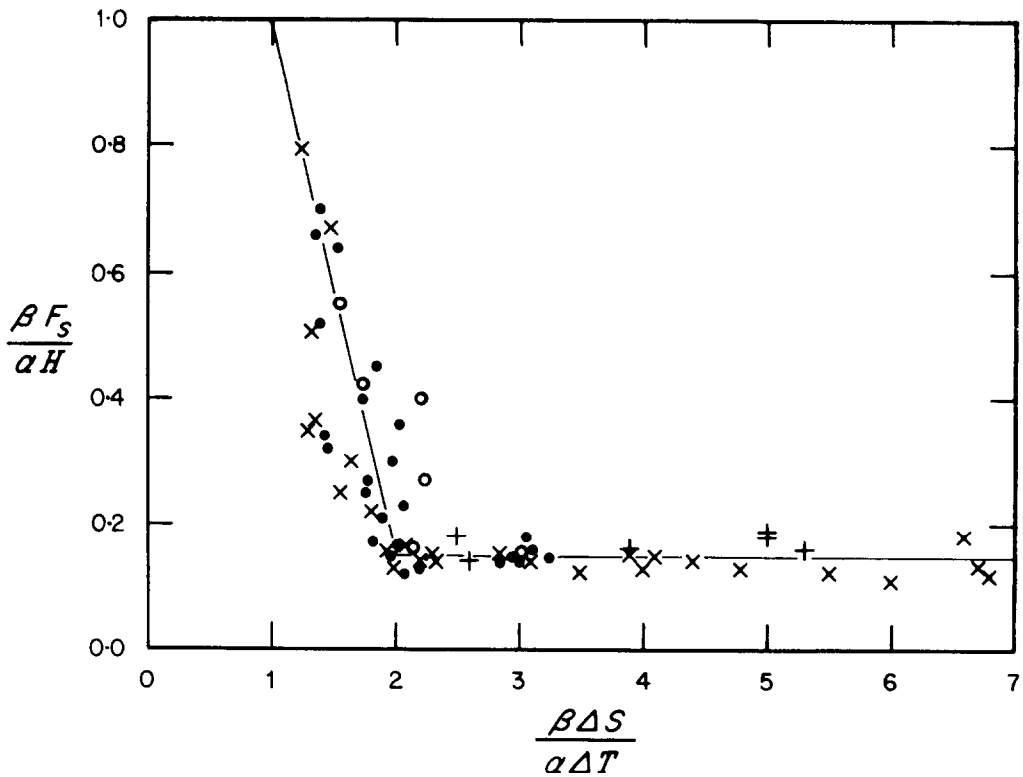


FIG. 7. The ratio of the potential energy changes due to the transfer of salt and heat across the interface, plotted as a function of stability.

interface expressed as the ratio of the contributions of salt and heat to the density difference. A closer examination of the experimentally determined functions f_1 and f_3 will now allow us to say more about the mechanism of the transfer processes taking place at the salinity-temperature interface. The detailed explanation of all features of the experimental curves remains, of course, a major theoretical problem.

Figure 4 shows that at small stabilities the heat flux is greater than it is over a solid plane with the same temperature contrast. This must be an effect of the wavy, breaking interface which brings elements with greater temperature contrasts into contact and increases the surface area. The values are greatest (and most scattered) just when the interface is about to break down completely. They fall to the "solid plane" value when the temperature difference compensates for about half the density difference due to salt, corresponding in our experiments to a net density difference of about one part per thousand. At higher density ratios the heat flux is below the value appropriate to a solid surface, so that the lifting of salt by the thermally driven motion is definitely inhibiting the transfer of heat.

The ratio of the transfer coefficients in Fig. 5 shows a similar decrease with increasing density ratio (or density difference). At small stabilities K_S/K_T approaches unity, suggesting that heat and salt are being transported by the same turbulent eddies. At larger, but still small, values of $\beta\Delta S/\alpha\Delta T$ the ratio K_S/K_T falls rapidly. This behavior is understandable qualitatively if we consider what happens to an individual turbulent element or wave at the interface. This can transfer heat by conduction, while little salt is transferred until breaking of the wave occurs, followed by intimate molecular mixing. The tendency to break will decrease as the stability increases, and the relative rate of transfer of salt will thus be reduced in the way which has been observed. Similar arguments are familiar in the context of stable shear flows, where the rate of transfer of heat or salt is reduced relative to momentum as the stability increases [4]. In that case momentum can be transported by pressure forces, while heat or salt diffusion depends on detailed mixing. The present experiments again raise the question of the possible influence of molecular

diffusivities on turbulent transports, as suggested by Batchelor and Townsend [5].

For still larger values of $\beta\Delta S/\alpha\Delta T$ the rate of decrease of K_S/K_T with increasing stability becomes much slower. At the highest stabilities studied, the ratio K_S/K_T approaches the molecular ratio k_S/k_T , which at the temperature appropriate to this run is about 1.5×10^{-2} . In these latter experiments an intermediate layer of depth about $\frac{1}{2}$ cm was built up, in which the motion was still disordered but very much damped. At even greater stabilities it seems likely that turbulence would be suppressed completely in this intermediate layer and the transfers effected by molecular diffusion alone, but the present experiments do not rule out the possibility that some kind of weak turbulent motion is always present under these circumstances.

Finally let us consider Fig. 7, which contains the most striking and clear cut result of these experiments. The ratio $\beta F_S/\alpha H$ at first falls rapidly as the stability is increased. At

$$\beta\Delta S/\alpha\Delta T = 2$$

it becomes constant, within the accuracy of our experiments, for all greater stabilities, with a mean value $\beta F_S/\alpha H = 0.15$. The theoretical implications of this result have not yet been fully explored, but it has a definite physical meaning: at all stabilities large enough for the salt flux to be reducing the heat flux, a constant fraction of the potential energy released by the temperature field is used to lift the salt. This suggests that there must be some self-limiting mechanism controlling the coupled diffusion of salt and heat which can operate over a large range of density differences, nearly 100:1 in our experiments.

ACKNOWLEDGEMENTS

I am grateful to Karl Schleicher for lending me the thermistor and conductivity bridges which he developed, and to Melvin Stern, Henry Stommel and George Veronis for many helpful discussions. This is Contribution No. 1541 from the Woods Hole Oceanographic Institution, and was supported in part by NSF Grant No. GP-317.

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Résumé—Lorsqu'une couche d'eau douce froide se trouve au-dessus d'eau salée chaude avec une interface mince entre elles, le système dans son ensemble peut être instable, même si les différences de densité $\beta\Delta S$ dues à la salinité sont beaucoup plus grandes que celles dues à la température $a\Delta T$. Les mouvements de convection sont produits dans les deux couches par le transport de chaleur, et en même temps le sel est transporté à travers l'interface. On présente ici les mesures des vitesses de transport dans un tel système à deux couches dans lequel la différence de température est maintenue en chauffant continuellement par en-dessous.

On a trouvé qu'à la fois le transport de chaleur et le rapport des vitesses de transport turbulent de sel et de chaleur dépendent systématiquement du rapport de densités $\beta\Delta S/a\Delta T$, au moins dans la faible gamme de flux de chaleur employée dans ces expériences. Ceci est en accord avec le résultat d'une analyse dimensionnelle simple, et suggère que la forme des fonctions auraient une application plus large. Les mesures indiquent aussi que dans une gamme large de différences de densités, le rapport du changement d'énergie potentielle de la couche supérieure due à l'ascension du sel est une fraction constante de celle libérée par le transport de chaleur.

Zusammenfassung—Liegt eine Schicht frisches kaltes Wasser über heissem Salzwasser mit einer scharfen Trennlinie dazwischen, so kann das System als Ganzes instabil sein, auch wenn die Dichteunterschiede $\beta\Delta S$ wegen des Salzgehaltes weitaus grösser sind als die durch die Temperatur $a\Delta T$ hervorgerufenen. In beiden Schichten werden durch den Wärmetransport Konvektivbewegungen hervorgerufen und gleichzeitig wird Salz über die Trennlinie transportiert. Für die Transportgeschwindigkeiten in einem solchen Zweischichtensystem, wo die Temperaturdifferenz durch kontinuierliche Beheizung von unten aufrechterhalten wird, werden hier Messungen angeführt. Es ergab sich, dass sowohl der Wärmeübergang wie auch das Verhältnis der turbulenten Transportgeschwindigkeiten von Salz und Wärme systematisch von dem Dichteverhältnis $\beta\Delta S/a\Delta T$ abhängen, zumindest für den kleinen Bereich der Wärmestromdichten, die bei diesen Versuchen angewandt wurden.

Dies steht in Übereinstimmung mit dem Ergebnis einer einfachen Dimensionsanalyse und legt nahe, dass die Form der Funktionen eine erweiterte Anwendbarkeit haben könnte. Die Messungen zeigen auch, dass das Verhältnis der potentiellen Energieänderung der oberen Schicht, die vom emporsteigenden Salz hervorgerufen wird, über einen weiten Bereich von Dichtedifferenzen ein konstanter Bruchteil des vom Wärmeübergang ausgelösten Verhältnisses ist.

Аннотация—Когда слой холодной свежей воды находится на поверхности горячей соленой воды и между ними существует резкая поверхность раздела, система в целом может быть нестабильной, даже если разность плотностей $\beta\Delta S$ из-за солености намного превышает разность плотностей, возникающую благодаря разности температур $a\Delta T$. В результате переноса тепла возникают конвективные движения в обоих слоях и в то же время происходит перенос соли через поверхность раздела. Представлены измерения скоростей переноса в таких двухслойных системах, в которых разность температур поддерживается непрерывным подогревом снизу.

Найдено, что как перенос тепла, так и отношение скоростей турбулентного переноса соли и тепла систематически зависят от отношения плотностей $\beta\Delta S/a\Delta T$, по крайней мере в небольшом диапазоне тепловых потоков, использованных в данных опытах. Эти результаты согласуются с данными простого анализа размерностей и допускают, что форма этих функций имеет бы более широкое применение. Измерения также показали, что в большом диапазоне разности плотностей отношение изменения потенциальной энергии верхнего слоя благодаря подъему соли является постоянной долей энергии переноса тепла.